### COPYRIGHT RESERVED UG - BCA (F - 2001)

2022(New)

Time: 3 hours

Full Marks: 70



Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

#### Group - A

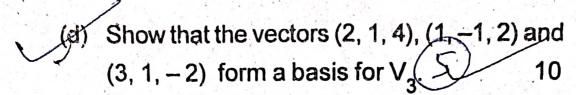
- 1. Answer any four questions of the following:
  - (a) Define subgroup of a group. Prove that a nonempty set H of a group G is a subgroup if and only if ∀ a, b ∈ H ⇒ ab<sup>-1</sup> ∈ H.
    - (b) Prove that every field is an integral domain.

10

(c) For a finite dimensional vector space V(F), any two bases have same number of elements.

EX - 71/2

(Turn over)



(e) Show that the following system of linear equations is not consistent:

$$x-4y+7z=14$$
  
 $3x+8y-2z=13$   
 $7x-8y+26z=5$ 

Verify the Caley Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$  10

- (g) Prove that the points (0, -1, 0), (2, 1, -1), (1, 1, 1) and (1, 2, 4) are coplanar. 10
- (h) Find the equation of the plane through the point (-1, 3, 2) and perpendicular to the planes x + 2y + 3z = 5 and 3x + 3y + z = 9.

10

### Group - B

Answer all questions of the following:

2. In the multiplicative group  $G = \{1, \omega, \omega^2\}$ , find the order of  $\omega$  and  $\omega^2$ ,

where 
$$\omega = \frac{-1 + \sqrt{3}i}{2}$$
.

EX - 71/2

(2)

Contd.

In the multiplicative group  $G = \{1, -1, i, -i\}$ , find the order of each element of G Define subgroup of a group. 3 5 Prove that in a ring R, (-a) (-b) = ab;  $\forall$  a, b  $\in$  R. 3 Define an integral domain. Write down a basis for R<sup>3</sup>. 3 State the elementary row operations ja a matrix. Define a matrix polynomial. Prove that the points (0, 4, 1), (2, 3, -1), (4, 5, 0) and (2, 6, 2) are the vertices of a square. 11. Define direction cosine of a line.

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## 2021

Time: 3 hours

Full Marks: 70

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The figures in the margin indicate full marks.

Answer any four questions in which Q. No. 1 is compulsory.

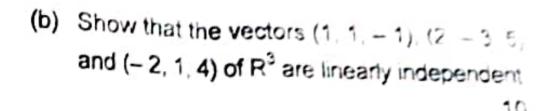
1. Answer all questions :

 $2 \times 10 = 20$ 

- (a) In the multiplicative group  $G = (1, \omega, \omega^2)$ , find the inverse of  $\omega^2$ .
- (b) In the multiplicative group G = (1, − 1, i, − i), find the order of i.
- (c) State Lagrange's theorem.
- (d) Prove that in a ring R, a (-b) = -ab; ∀ab∈ R.
- (e) Define a field.
- (f) Write down a basis for R4.

- (g) Write down a matrix of order three in . Z. Echelon form.
- (h) Define a matrix polynomial.
- (i) Determine the relation between Cartesian and Polar spherical coordinates of a point.
- (j) Find the direction cosines of a line passing through the points (4, 5, 0) and (2, 6, 2).
- 2. (a) Prove that in a group G, (ab)<sup>-1</sup> b<sup>-1</sup> a<sup>-1</sup>; ∀ a, b ∈ G. 10
  - (b) Prove that the identity element in a group is unique.
- (a) Prove that a finite integral domain is a field.
  - (b) Prove that the power set P(X) of a non-empty set X is a Boolean Algebra. with φ = 0 and X = 1 and with the operations Λ, ∨ and defined by A Λ B = A ∩ B; A ∨ B = A ∪ B and A' = X A, ∀ A, B ∈ P(X).
- (a) Prove that a subset W of a vector space V(F) is a subspace if W is closed under vector addition and scalar multiplication. 10

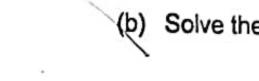
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(a) Reduce the matris :

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$
 to the normal form and find the rank.



(b) Solve the system of linear equations: 10

$$2x - y + 3z = 9$$
  
 $x + 3y - 2z = 1$   
 $5x - 3y + z = 2$ 

6. (a) Find the characteristic equation of the matrix:

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

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(b) Verify the Cayley Hamilton theorem for the matrix:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -1 & 1 & 4 \end{bmatrix}$$

- (a) Find the equation of a plane which cuts off the intercepts a, b, c from the coordinate axes.
  - (b) Find the equation of the plane passing through the point A(5, - 3, 2) and perpendicular to OA, where O is the origin.

10

8. (a) Prove that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$  are coplanar.

10

(b) Find the equation of the sphere whose center is (1, 2, -1) and which passes through the point (1, -2, 3).

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### 2019

Time: 3 hours

Full Marks: 70

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The figures in the margin indicate full marks.

Answer any four questions in which Q. No. 1 is compulsory.

Answer all the questions of the following :

$$1 \times 10 = 20$$

- (i) Form the composition table of the multiplicative group of cube roots of unity.
- (ii) Define the inverse of an element in a group.
- (iii) Define cosets and state Lagrange's theorem.
- (iv) Prove that in a ring R, a(–b) = –(ab); ∀a, b∈ R.
- · (v) Define linear independence of vectors.

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(Turn over)

- (vi) Define a basis of a vector space.
- (vii) State the condition for a system of linear equations to have no solution.
- (viii)Write down a matrix of order four in Eclon form.
- (ix) Determine the relation between Cartesian and polar spherical coordinates of a point.
- (x) Find the direction cosines of a line passing through the points (4,5,0) and (2,6,2).
- (a) Prove that in a group G<sub>i</sub>(ab)<sup>-1</sup> = b<sup>-1</sup>a<sup>-1</sup>;
   ∀ a, b ∈ G
  - (b) Prove that the identity element in a group is unique.10
- (a) Prove that a finite integral domain is a field.
  - (b) Prove that the power set P(X) of a non empty set X is a Boolean Algebra with φ = 0 and X = 1 and with the operations ∧, ∨ and defined by, A ∧ B = A ∩ B; A ∨ B = A ∪ B and A' = X − A, ∀ A, B ∈ P(X).

- (a) Prove that a subset W of a vector space V(F) is a subspace if W is closed under vector addition and scalar multiplication. 10
  - (b) Show that the vectors (1,1,-1), (2,-3,5) and (-2,1,4) of R<sup>3</sup> are linearly independent. 10
- 5. (a) Find the rank of the matrix

  10

  11 5 7 1

  13 12 1 1

  1-1 2 -13 1
  - (b) Solve the system of linear equations: 10 2x-y+3z=9x+3y-2z=1
- 6. (a) Find the eigen values of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & 4 & 0 \end{bmatrix}$
- (b) Write down the matrix of the quadratic form  $x_1^2 18x_1x_2 + x_2^2 \text{ and verify that it can be}$ written as matrix products X AX. 10

5x - 3y + z = 2

- 7. (a) Find the angle between two lines whose direction cosines are (I,m,n) and (I,m,n).
  - (b) Find the equation of the plane passing through the point A(5,-3,2) and perpendicular to OA, where O is the origin.
- Find the shortest distance between two skew lines.